ඌව පළාත් අධ්‍යාපන දෙපාර්තමේන්තුව

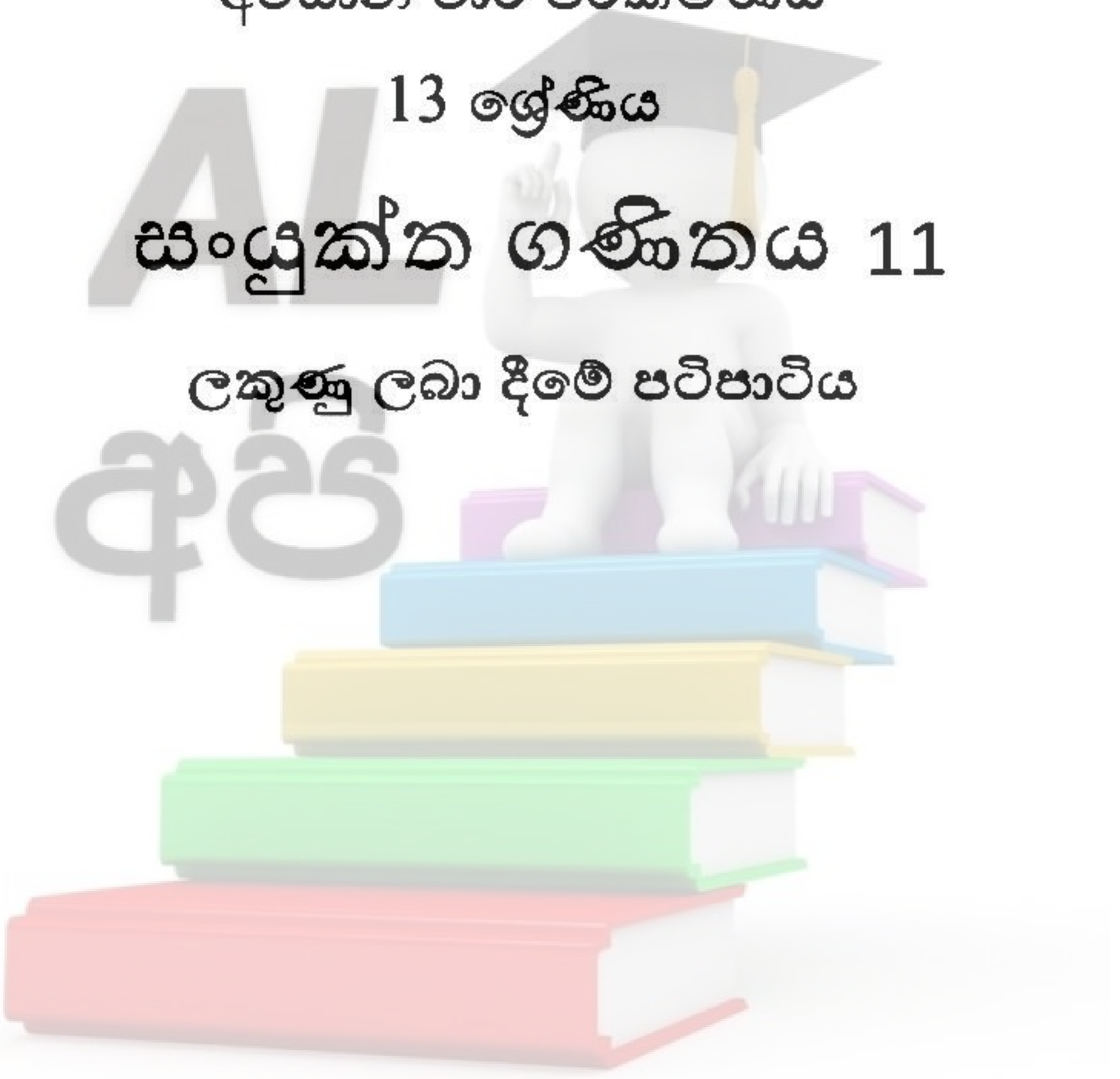
අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය 2026

අවසාන වාර පරීක්ෂණය

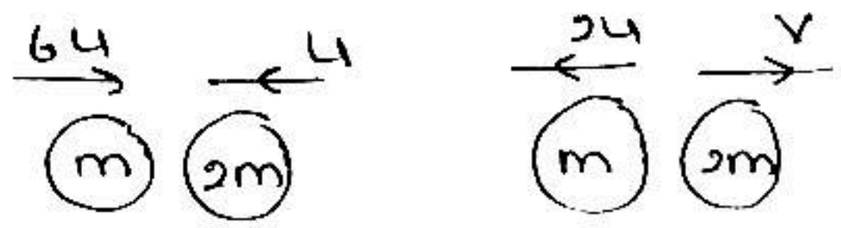
13 ශ්‍රේණිය

සංයුක්ත ගණිතය 11

ලකුණු ලබා දීමේ පටිපාටිය



(01)



$I = \Delta mv \longrightarrow$ සද්ධන්යෙව

$0 = m(-2u) + 2mv - [m(6u) + 2m(-u)]$ 5

$6u = 2v$

$v = 3u$ 5

ගැ.ප.නි.

$v + 2u = e(6u + u)$ 5

$3u + 2u = 7u.e$

$e = 5/7$ 5

ආරම්භක චා.ශ $T_1 = \frac{1}{2}m(6u)^2 + \frac{1}{2}.2mu^2 = 19mu^2$

අවසාන චා.ශ $T_2 = \frac{1}{2}m(2u^{(-)})^2 + \frac{1}{2}\ 2m(3u)^2 = 11mu^2$ 5

$T_2/T_1 = 11/19 \implies T_2 = 11/19\ T_1$

25

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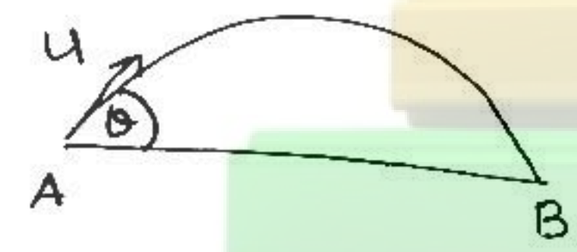
(02)



$\rightarrow\ s = ut + \frac{1}{2}at^2$

$R = u\cos\theta\ t$

$\uparrow\ s = ut + \frac{1}{2}at^2$ 5

$0 = u\sin\theta\ t - \frac{1}{2}gt^2$

$t \neq 0 \implies t = \frac{2u}{g}\sin\theta$

$\therefore R = u\cos\theta\ \frac{2u}{g}\sin\theta \implies R = \frac{u^2}{g}\sin 2\theta$ 5

$\theta = \alpha \implies R = R_1 \implies R_1 = \frac{u^2}{g}\sin 2\alpha$

$\theta = \beta \implies R = R_2 \implies R_2 = \frac{u^2}{g}\sin 2\beta$ 5

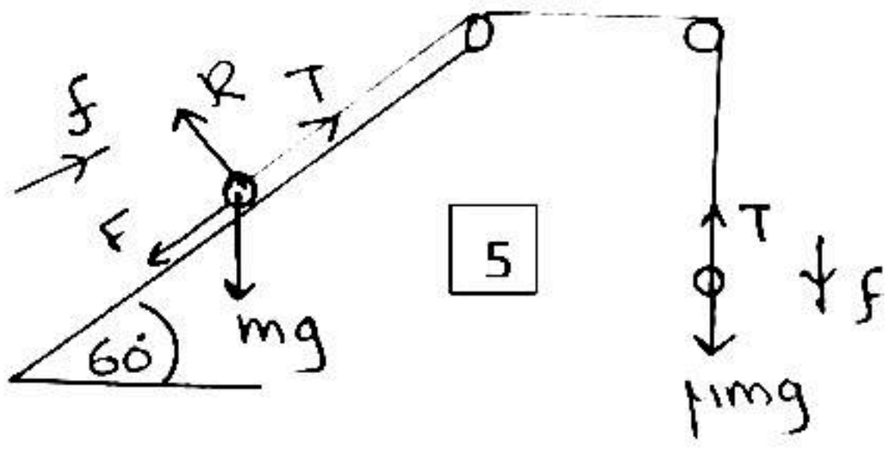
$R_1 + R_2 = \frac{u^2}{g}(\sin 2\alpha + \sin 2\beta) = 2\frac{u^2}{g}\sin(\alpha+\beta)\cos(\alpha-\beta)$

$R_1 - R_2 = \frac{2u^2}{g}\cos(\alpha+\beta)\sin(\alpha-\beta)$ 5

$\therefore \frac{R_1 + R_2}{R_1 - R_2} = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} \cdot \frac{\cos(\alpha-\beta)}{\sin(\alpha-\beta)} = \frac{\tan(\alpha+\beta)}{\tan(\alpha-\beta)}$ 5

25

(03)



5

m o ↗ 60°

$R = mg\cos 60^\circ$

$R = mg/2$

$F = \mu mg/2$ 5

$F = ma$; m o ↗ $T - mg\sin 60^\circ - F = mf$

$T - \frac{\sqrt{3}}{2}mg - \mu\frac{mg}{2} = mf$ —— ① 5

μmg o ↓ $\mu mg - T = \mu mf$ —— ② 5

① + ② $\frac{\mu mg}{2} - \frac{\sqrt{3}}{2}mg = f(m+\mu m)$

5

$f > 0$ විය යුතුය. එවිට $\mu > \sqrt{3}$

25

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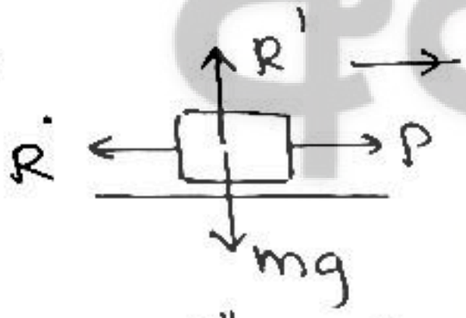
(04)

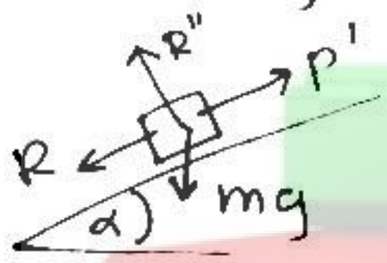


$H = PV$

$H \times 10^3 = PV$

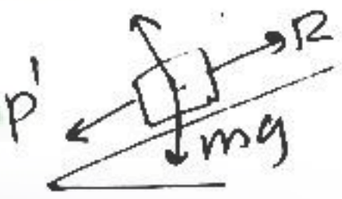
$R = P$

$= \frac{H \times 10^3}{V}$ 5



$10^3 \times H - P'\frac{V}{2}$

$P' = \frac{2H \times 10^3}{V}$ 5



$F = ma$

$\frac{2 \times 10^3 H}{V} - R - mg\sin\alpha = ma_1$

$\frac{2 \times 10^3}{V}H - \frac{H \times 10^3}{V} - mg\sin\alpha = ma_1$

$a_1 = \frac{H \times 10^3}{mV} - g\sin\alpha$ 5

$\frac{2 \times 10^3 H}{V} - R + mg\sin\alpha = ma_2$

$\frac{2 \times 10^3}{V}H - \frac{H \times 10^3}{V} + mg\sin\alpha = ma_2$

$a_2 = \frac{H \times 10^3}{mV} + g\sin\alpha$ 5

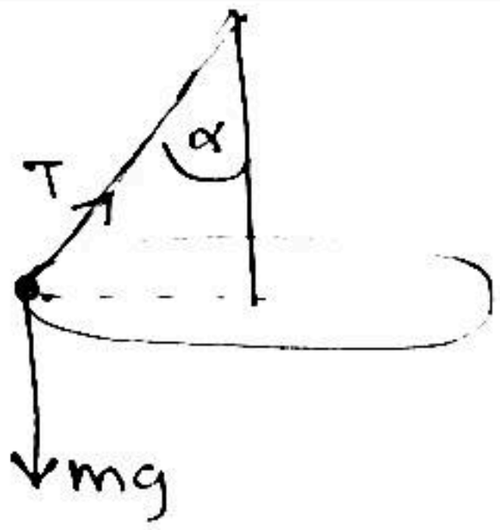
5

$2a_1 = a_2 \Rightarrow 2\frac{H \times 10^3}{mV} - 2g\sin\alpha = \frac{H \times 10^3}{mV} + g\sin\alpha$

5 $3g\sin\alpha = \frac{H \times 10^3}{mV} \Rightarrow \sin\alpha = \frac{H \times 10^3}{3mgV}$

25

(05)



$\uparrow T\cos\alpha = mg$

$T = mg/\cos\alpha$ [5]

$\longrightarrow F = ma$

$T\sin\alpha = m\dfrac{a^2\omega^2}{a}$ [5]

$T\sin\alpha = m(l\sin\alpha)\omega^2$

$\dfrac{mg}{\cos\alpha} = ml\omega^2$ [5]

$\Rightarrow \omega^2 = \dfrac{g}{l\cos\alpha} \Rightarrow \omega = \sqrt{\dfrac{g}{l\cos\alpha}}$ [5]

$T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{l\cos\alpha}{g}}$ [5]

25

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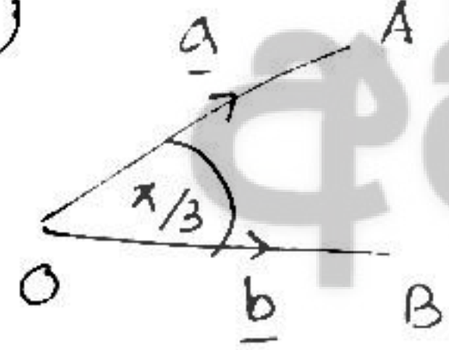
(06)



$\underline{a} = \underline{i} + 2\underline{j}$ $\quad \underline{b} = x\underline{i} + y\underline{j}$

$|\underline{a}| = \sqrt{5}$ $\quad |\underline{b}| = 2\sqrt{5}$

$\sqrt{x^2+y^2} = 2\sqrt{5}$

$x^2+y^2 = 20$ [5]

$\underline{a}\cdot\underline{b} = |\underline{a}||\underline{b}|\cos\pi/3$

$(\underline{i}+2\underline{j})\cdot(x\underline{i}+y\underline{j}) = \sqrt{5}\cdot 2\sqrt{5}\cdot \frac{1}{2}$ [5]

[5] $x + 2y = 5 \Rightarrow x = 5-2y$

$\therefore (5-2y)^2 + y^2 = 20$

$25 - 20y + 4y^2 + y^2 = 20$

$5y^2 - 20y + 5 = 0$

$y^2 - 4y + 1 = 0$

$y = \dfrac{4 \pm \sqrt{16-4}}{2}$

$y = 2 \pm \sqrt{3}$ [5]

(+) $x = 1 - 2\sqrt{3} < 0$

(−) $x = 1 + 2\sqrt{3} > 0$

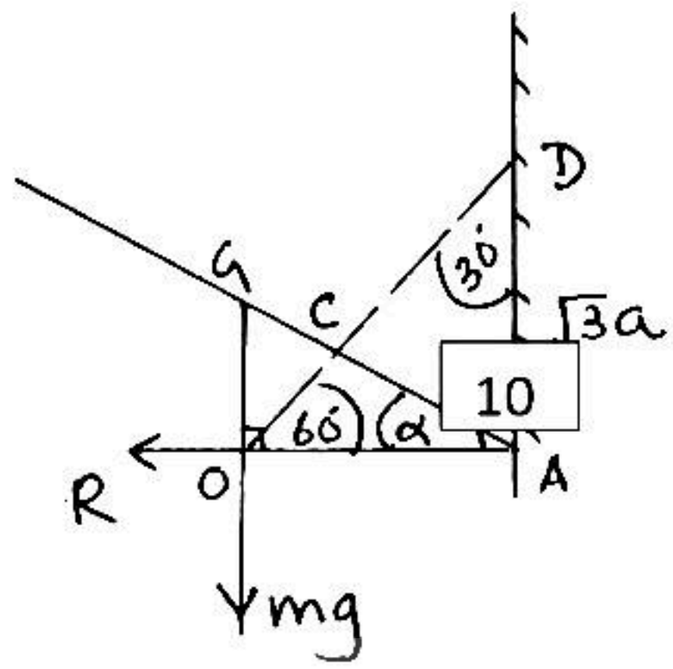
$\therefore y = 2+\sqrt{3}$

$x = 1 - 2\sqrt{3}$ [5]

$\because x < 0$

25

07



$\tan 30° = \frac{OA}{\sqrt{3}a}$ 5

$OA = a$

$AG \cos\alpha = OA$ 5

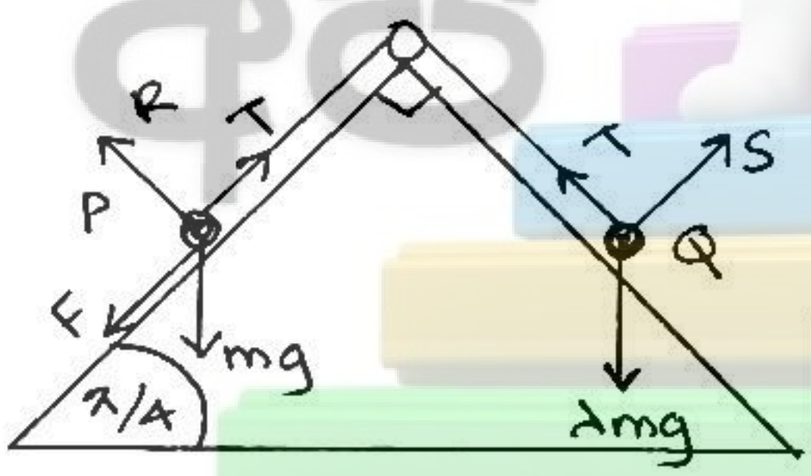
$4a \cos\alpha = a$

$\cos\alpha = \frac{1}{4}$ 5

$\alpha = \cos^{-1}(\frac{1}{4})$

25

08



5

P ↑ $R = mg\cos\frac{\pi}{4}$

$R = \frac{mg}{\sqrt{2}}$ 5

Q ↖ $T = \lambda mg \cos\frac{\pi}{4}$

$T = \frac{\lambda mg}{\sqrt{2}}$ 5

P ↗ $F = T - mg\cos\frac{\pi}{4}$

$= \frac{\lambda mg}{\sqrt{2}} - \frac{mg}{\sqrt{2}}$

$= \frac{(\lambda-1)}{\sqrt{2}} mg$.

$\frac{|F|}{R} \leq \mu$ 5

$|\lambda - 1| \leq \frac{1}{3}$

$-\frac{1}{3} \leq \lambda - 1 \leq \frac{1}{3}$ 5

$\frac{2}{3} \leq \lambda \leq \frac{4}{3}$

25

(09) $P(B'/A) = P(B/A)$

$$\frac{P(B' \cap A)}{P(A)} = \frac{P(B \cap A)}{P(A)}$$

$\Rightarrow P(B \cap A) = P(B' \cap A)$ 5

$P(A \cap B') = P = P$

$P(A \cup B) = P(\Omega) = 1$

$P(A \cap B') + P(A \cap B) + P(A' \cap B) = 1$ 5

$P + P + P(A' \cap B) = 1$

$P(A' \cap B) = 1 - 2P$ 5

$P(A' \cap B) + P(A \cap B) = P(B)$ 5

$1 - 2p + p = 2/3$

$p = 1 - 2/3 = 1/3$ 5

25

(10) මාතය 5, මධ්‍යස්ථය 5, මධ්‍යන්‍යය 6/5

x_1, x_2, x_3, x_4, x_5

$x_3 = x_4 = 5$, $x_1 = x_2 = a$

$x_5 = a + 3$ 5

$a \quad a \quad 5 \quad 5 \quad a+3$

$\bar{x} = \frac{13 + 3a}{5}$ 5

$$\frac{a^2 + a^2 + 2(5^2) + (a+3)^2}{5} - \left(\frac{3a+13}{25}\right)^2 = \frac{6}{5}$$ 5

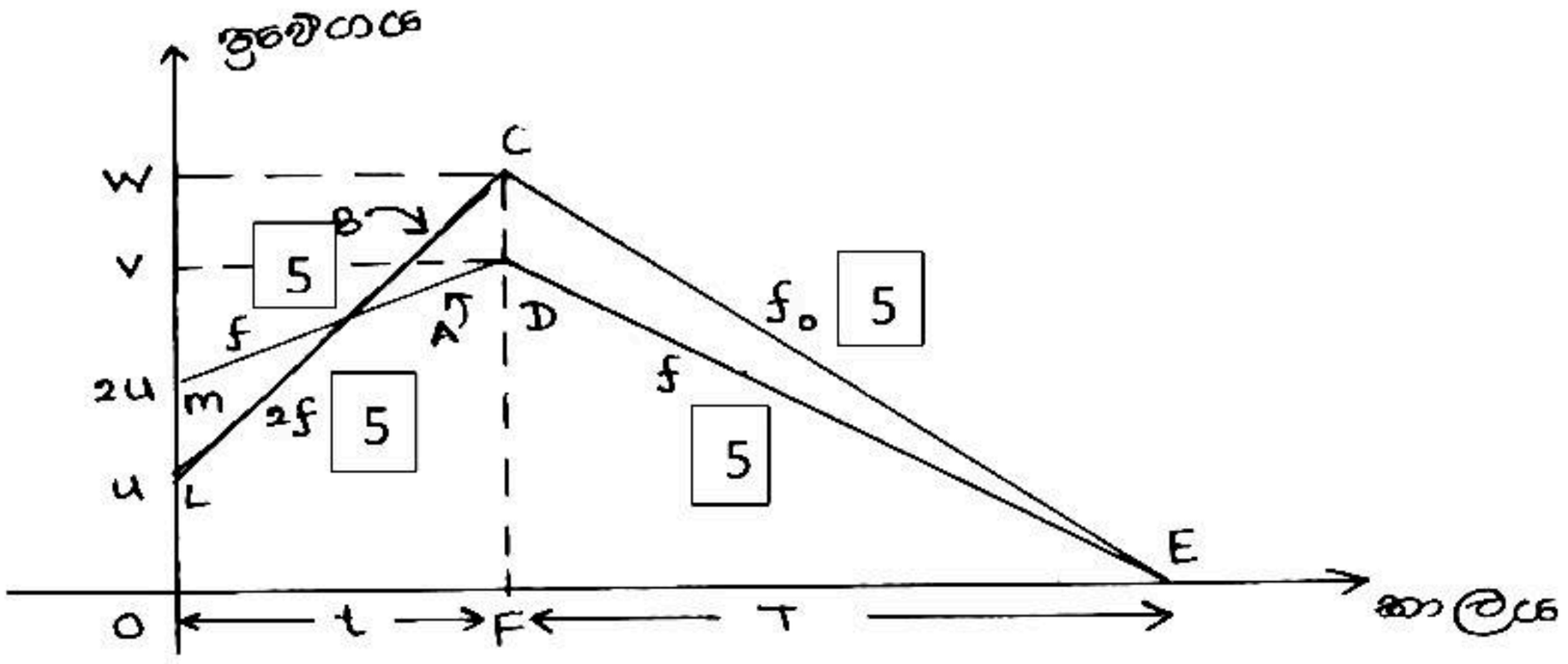
$6a^2 - 48a + 96 = 0$ ∴ 4, 4, 5, 5, 7

$a^2 - 8a + 16 = 0$ 5

$(a-4)^2 = 0 \Rightarrow a = 4$ 5

25

(11)



20

P → Q ට චලිතය

A/ $f = \frac{V-2u}{t}$ —— ① 5

B/ $2f = \frac{W-u}{t}$ —— ② 5

A හි විස්තාපනය = B හි විස්තාපනය

OMDF □ ව.ඒ = OLCF □ ව.ඒ

$\frac{1}{2}(2u+V)t = \frac{1}{2}(u+W)t$ 5

$2u+V = u+W$

$W-V = u$ —— ③

① ත් $V = 2u + ft$

② ත් $W = u + 2ft$

$u + 2ft - 2u - ft = u$

$ft = 2u$

$t = \frac{2u}{f}$ 5

A හි ප්‍රවේගය $= 2u + f\frac{2u}{f} = 4u$ 5

B හි ප්‍රවේගය $= u + 2f\frac{2u}{f} = 5u$ 5

Q → R චලිතයට

$f_0 = \frac{W}{T} = \frac{5u}{T} \Rightarrow T = \frac{5u}{f_0}$ 5

35

QR දුර = T කාලය තුළ B හි විස්ථාපනය

$= \frac{1}{2} \times W \times T$

$= \frac{1}{2} \times 54 \times \frac{54}{f_0}$ 5

$= \frac{254^2}{2f_0}$ —— ④

Q → R චලිතයට

A/ $f = \frac{V}{T} \Rightarrow T = \frac{44}{f}$ 5

A හි විස්ථාපනය = QR දුර

$= \frac{1}{2} \times V \times T$

$= \frac{1}{2} \times 44 \times \frac{44}{f}$

$= \frac{84^2}{f}$ —— ⑤ 5

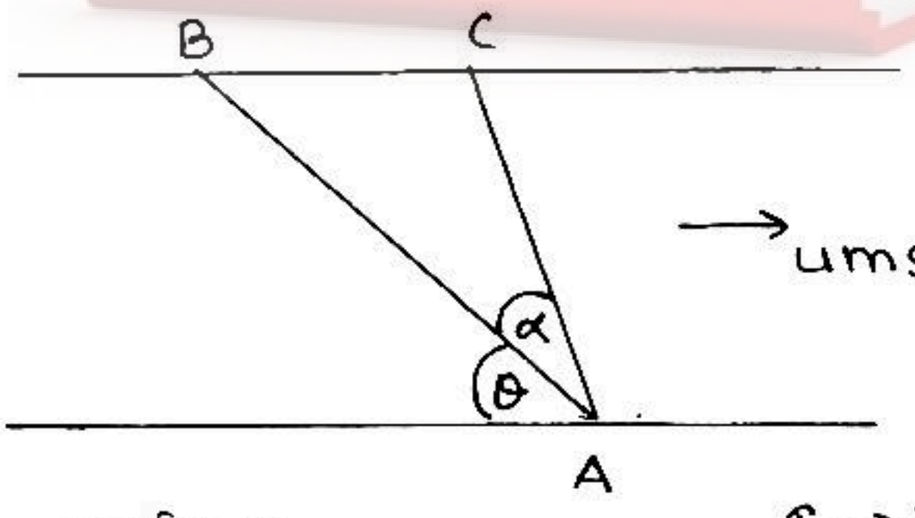
④ = ⑤ ද?

$\frac{84^2}{f} = \frac{254^2}{2f_0}$ 5

$16f_0 = 25f$

20

b)



බෝට්ටුව - M

ගඟ - W

නොළුව - E

A → C චලිතය

$\underline{V}(W,E) = \rightarrow u$

$\underline{V}(M,W) =$ 

$\underline{V}(M,E) =$ 

5

C → B චලිතය

$\underline{V}(M,W) = \leftarrow V$

$\underline{V}(M,E) = \twoheadleftarrow$

A → C

$\underline{V}(m,E) = \underline{V}(m,W) + \underline{V}(W,E)$

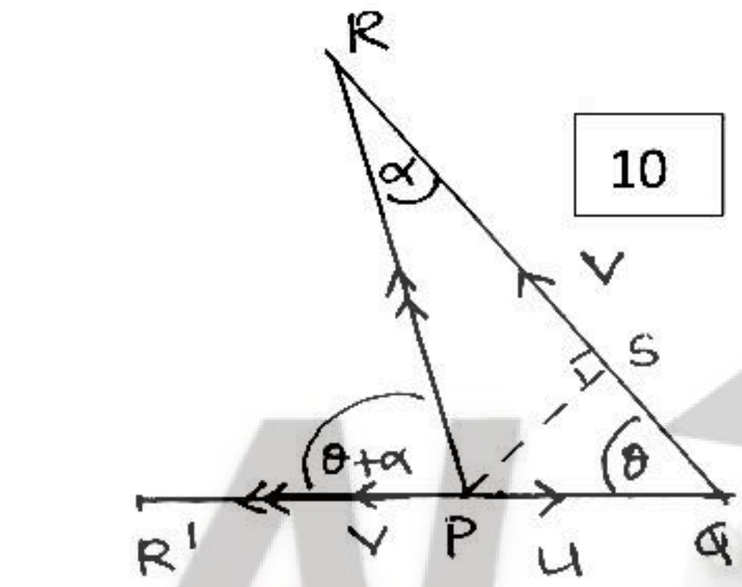
$\overrightarrow{PR} = \overrightarrow{QR} + \overrightarrow{PQ}$

C → B

$\underline{V}(m,E) = \underline{V}(m,W) + \underline{V}(W,E)$

$\overrightarrow{PR'} = \overrightarrow{QR'} + \overrightarrow{PQ}$

5



10

$PR^2 = U^2\sin^2\theta + V^2 + U^2\sin^2\theta - 2UV\cos\theta$

$PR = \sqrt{U^2 + V^2 - 2UV\cos\theta}$

5

PRS Δ ඉන් $\tan\alpha = \dfrac{U\sin\theta}{V - U\cos\theta}$

5

$\therefore \sin\alpha = \dfrac{U\sin\theta}{\sqrt{U^2+V^2-2UV\cos\theta}}$ —①

5

35

KAC = BC

ABC Δ ට සයින් නියමයෙන්

$\dfrac{AC}{\sin\theta} = \dfrac{BC}{\sin\alpha}$

$\dfrac{AC}{\sin\theta} = \dfrac{K.AC}{\sin\alpha}$

5

$\therefore \sin\alpha = K\sin\theta$ —②

① = ②

$K\sin\theta = \dfrac{U\sin\theta}{\sqrt{U^2+V^2-2UV\cos\theta}}$

5

$K = \dfrac{U}{\sqrt{U^2+4U^2-4U^2\cos\theta}}$

5

$K = \dfrac{1}{\sqrt{5-4\cos\theta}}$

$K^2 = 1/(5-4\cos\theta)$

$\cos\theta = \dfrac{5K^2-1}{4K^2}$

5

$0 < \theta < \pi/2$ නිසා

$0 < \cos\theta < 1$

$0 < \dfrac{5K^2-1}{4K^2} < 1$

5

$0 < 5K^2 - 1 < 4K^2$

$5K^2 > 1$ සහ $K^2 - 1 < 0$

$K < -\frac{1}{\sqrt{5}}$ හෝ $\frac{1}{\sqrt{5}} < K$ සහ $-1 < K < 1$

5

$K > \frac{1}{\sqrt{5}}$

5

නමුත් $K > 0$

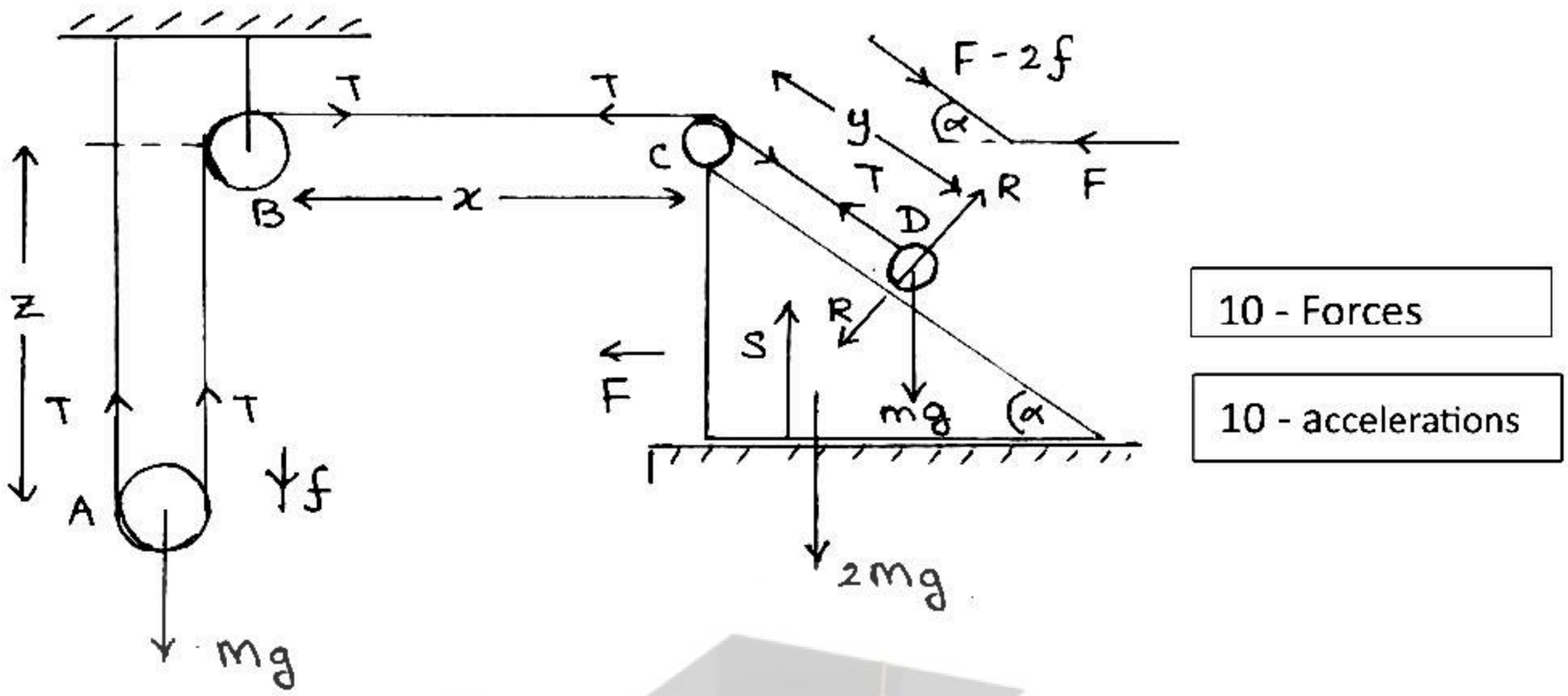
$\Rightarrow \frac{1}{\sqrt{5}} < K < 1$

5

$\therefore 1 < \sqrt{5}K < \sqrt{5}$

40

(12) a)



10 - Forces

10 - accelerations

$x + y + 2z + k = l$

$\ddot{x} + \ddot{y} + 2\ddot{z} = 0$ 10

$\ddot{x} = -F$

$\ddot{z} = f$ නම් $\ddot{y} = F - 2f$ 5

$F = ma$ යොදමු.

A ට $\downarrow$; $Mg - 2T = Mf$ — ① 10

D ට $\searrow$; $mg\sin\alpha - T = m(F - 2f - F\cos\alpha)$ — ② 10

2M සහ m ට

$\leftarrow$; $T = 2MF + m[F - (F - 2f)\cos\alpha]$ — ③ 10

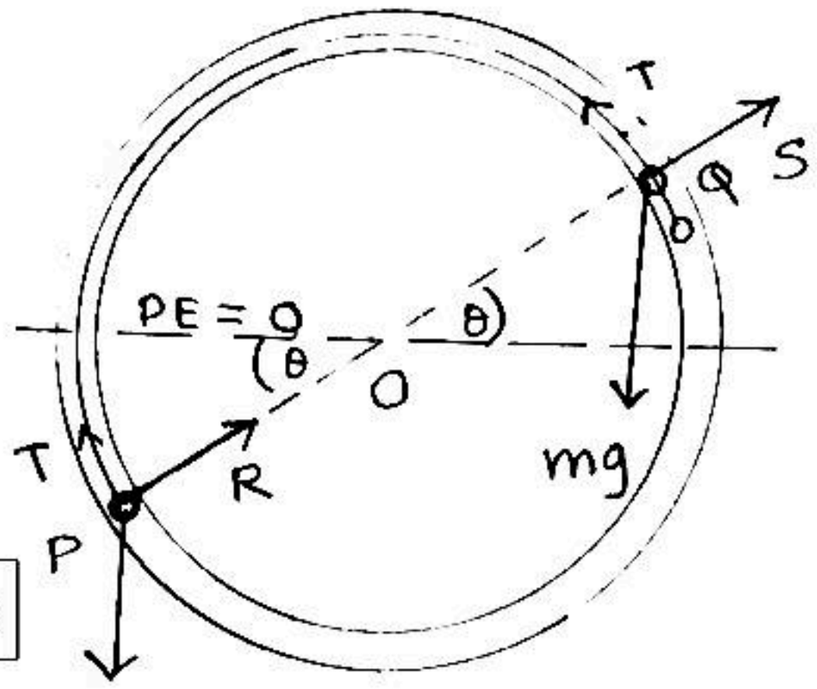
$f = F$ බව නමේ

② + ③ න් $mg\sin\alpha = mF - 2mF - mF\cos\alpha + 2MF + mF - mF\cos\alpha + 2mF\cos\alpha$ 10

$$F = \frac{mg\sin\alpha}{2M}$$ 5

80

(12) b)



5

$$0 + 0 = -2mg\sin\theta\, a + \frac{1}{2}\cdot 2mV^2 + mg\,a\sin\theta + \frac{1}{2}mV^2$$ 20

$$0 = -4ga\sin\theta + 2ga\sin\theta + 3V^2$$

$$3V^2 = 2ga\sin\theta$$

$$V^2 = \frac{2}{3}ga\sin\theta$$ 5

$\theta = \sin^{-1}(3/4)$ ලබා ගැනීම

$$V^2 = \frac{2}{3}ga \times \frac{3}{4}$$ 5

$$V^2 = \frac{ga}{2} \Rightarrow V = \sqrt{\frac{ga}{2}}$$ 5

↙ $F = ma$

$$-2mg\sin\theta + R = 2m\frac{V^2}{a}$$ 5

$$R = \frac{2m}{a}\,\frac{ga}{2} + 2mg\cdot\frac{3}{4}$$

$$= \frac{5mg}{2}$$ 5

Q ට ශ.ඤ. නි. $mga\sin\theta + \frac{1}{2}mV_1^2 = mga + \frac{1}{2}mV_1^2$ 10

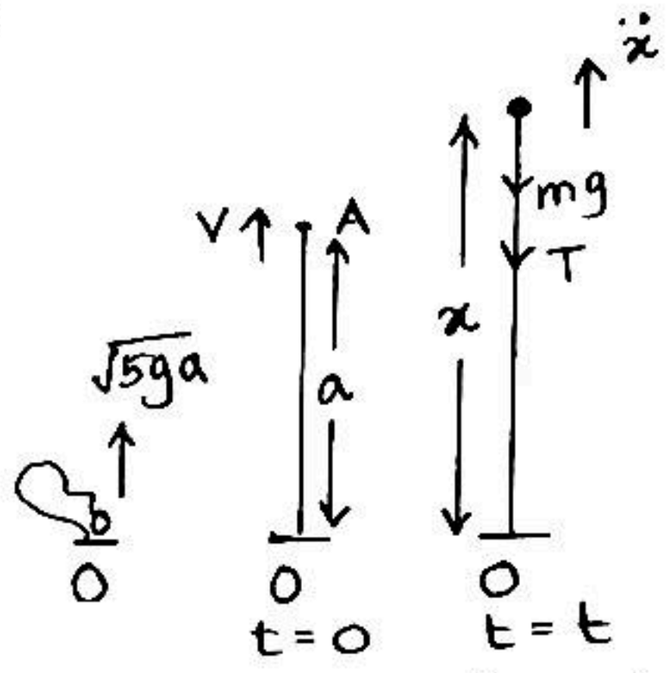
$$2ga \times \frac{3}{4} + g\frac{a}{2} = 2ga + V_1^2$$

$$V_1^2 = 0 \Rightarrow V_1 = 0$$ 5

∴ ඉහළම ලක්ෂයට යන්නෙන් ලත්තා වේ 5

80

(13)



$O \to A$ ඇ $v^2 = u^2 + 2gh$

$v^2 = 5ga - 2ga$ 5

$v = \sqrt{3ga}$ 5

$T = mg\dfrac{(x-a)}{a}$ 5

$\uparrow F = ma$

$-mg - T = m\ddot{x}$ 5

$-mg - mg\dfrac{x}{a} + mg = m\ddot{x}$ 5

$\ddot{x} + \dfrac{g}{a}x = 0$ — (1) 5

30

$x = \alpha \cos\omega t + \beta \sin\omega t$

$t = 0$ වීට $x = a$

$a = \alpha \cos 0 + \beta \sin 0$ 5

$\Rightarrow \alpha = a$ 5

$\dot{x} = -\alpha\omega \sin\omega t + \beta\omega \cos\omega t$ 5

$t = 0$ වීට $\dot{x} = \sqrt{3ga}$ 5

$\sqrt{3ga} = \beta\omega$

$\ddot{x} = -\alpha\omega^2 \cos\omega t - \beta\omega^2 \sin\omega t$ 5

$\ddot{x} = -\omega^2(\alpha \cos\omega t + \beta \sin\omega t)$

$\ddot{x} = -\omega^2 x$ 5

$\ddot{x} + \omega^2 x = 0$ — (2)

(1) හා (2) සැසඳීමෙන්

$\omega = \sqrt{g/a}$

$\therefore \beta = \sqrt{3}a$ 5

35

$\ddot{x} = 0$ වීට $x = 0$

$\therefore$ දෝලන කේන්ද්‍රය O හි පවතී. 5

$x = a$ වීට $V = \sqrt{3ga}$.

$v^2 = \omega^2(d^2 - x^2)$

$3ga = \dfrac{g}{a}(c^2 - a^2)$ 5

$c^2 = 4a^2$

$c = 2a$ 5

$\therefore$ $x = 2a$ වීට විස්තාරය දක්වයි. 5

20

O සිට $x = a$ දක්වා ↑ කාලය t_1 නම්

$\uparrow v = u + at$

$\sqrt{3ga} = \sqrt{5ga} - g t_1$ 5

$\Rightarrow t_1 = \sqrt{\frac{a}{g}} (\sqrt{5} - \sqrt{3})$ 5

$x = a$ සිට $x = 2a$ දක්වා කාලය t_2 ගනිමු.

$x = \alpha \cos \omega t + \beta \sin \omega t$ 5

$2a = a \cos \omega t_2 + \sqrt{3} \sin \omega t_2$

$1 = \cos (\omega t_2 - \pi/3)$ 5

$\cos (\omega t_2 - \pi/3) = \cos 0$ 5

$\Rightarrow \omega t_2 = \pi/3 \Rightarrow t_2 = \sqrt{\frac{a}{g}} (\pi/3)$ 5

5

$\therefore$ මුළු කාලය $= 2 (t_1 + t_2)$ 5

$= 2 \sqrt{\frac{a}{g}} \left(\pi/3 + \sqrt{5} - \sqrt{3} \right)$ 5

උපරිම උසේ දී $x = 2a$ 5

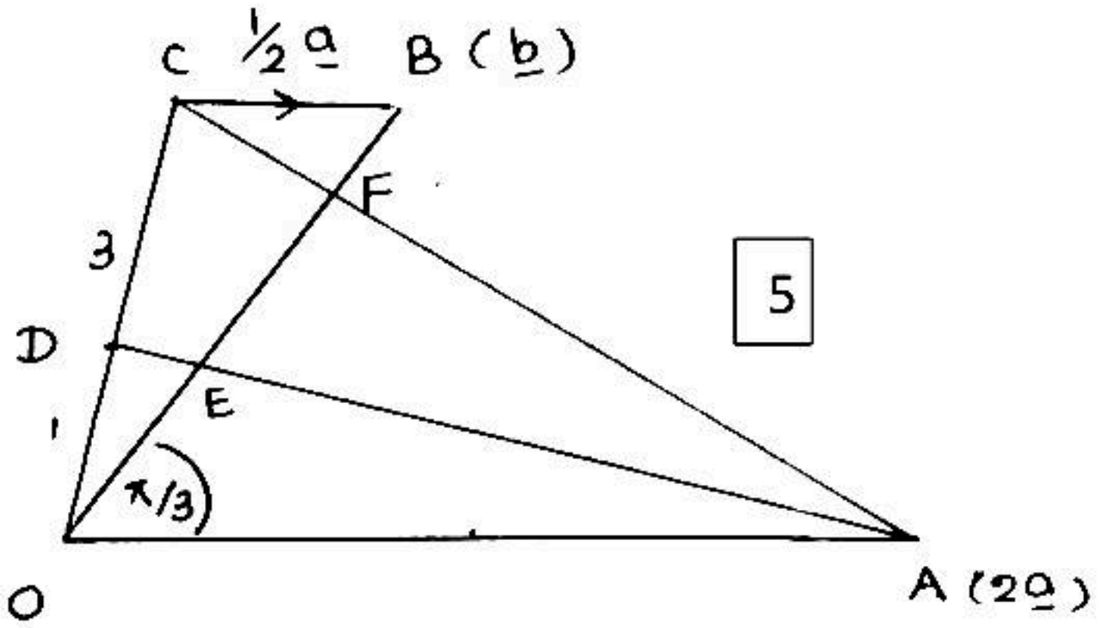
තන්තුවේ විතතිය $= 2a - a = a$ 5

$\therefore T_{x=2a} = mg \cdot \frac{a}{a}$ 5

$= mg$ 5

65

(14) a)



5

$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$

$= \underline{b} - \frac{1}{2}\underline{a}$ 5

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$

$= -2\underline{a} + \frac{1}{4}\underline{b} - \frac{1}{8}\underline{a}$

$= -\frac{17}{8}\underline{a} + \frac{1}{4}\underline{b}$ 5

$\overrightarrow{AE} = \lambda \overrightarrow{AD}$

$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$

$\overrightarrow{OE} = 2\underline{a} + \lambda\left(-\frac{17}{8}\underline{a} + \frac{1}{4}\underline{b}\right)$ —①

5

$\overrightarrow{OE} = \gamma \overrightarrow{OB}$

$= \gamma \underline{b}$ —③

5

$\overrightarrow{OD} = \frac{1}{4}\overrightarrow{OC}$

$= \frac{1}{4}\underline{b} - \frac{1}{8}\underline{a}$ 5

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$= -2\underline{a} + \underline{b} - \frac{1}{2}\underline{a}$

$= -\frac{5}{2}\underline{a} + \underline{b}$ 5

25

$\overrightarrow{FB} = \mu \overrightarrow{OB}$

$\overrightarrow{FC} = \overrightarrow{FB} + \overrightarrow{BC}$

$= \mu\underline{b} - \frac{1}{2}\underline{a}$ —② 5

$\overrightarrow{FC} = \beta \overrightarrow{AC}$

$= \beta\left(-\frac{5}{2}\underline{a} + \underline{b}\right)$ —④ 5

① = ③ $\Rightarrow 2\underline{a} + \lambda\left(-\frac{17}{8}\underline{a} + \frac{1}{4}\underline{b}\right) = \gamma\underline{b}$ 5

② = ④ $\Rightarrow \mu\underline{b} - \frac{1}{2}\underline{a} = \beta\left(-\frac{5}{2}\underline{a} + \underline{b}\right)$ —⑤ 5

$\underline{a}, \underline{b} \neq \underline{0}$ හා $\underline{a} \not\parallel \underline{b}$ නිසා

$2 - \frac{17}{8}\lambda = 0 \Rightarrow \lambda = 16/17$

$\frac{\lambda}{4} = \gamma \Rightarrow \gamma = 4/17$

$\frac{5\beta}{2} = \frac{1}{2} \Rightarrow \beta = 1/5$ 5

$\mu = \beta \Rightarrow \mu = 1/5$ 5

40

$A\hat{O}D = \pi/3 + \theta$; $\theta = B\hat{O}C$

$\overrightarrow{OC} = \underline{b} - \frac{1}{2}\underline{a}$ 5

$|\overrightarrow{OC}|^2 = \left(\underline{b} - \frac{1}{2}\underline{a}\right).\left(\underline{b} - \frac{1}{2}\underline{a}\right)$

$= |\underline{b}|^2 + \frac{1}{4}|\underline{a}|^2 - \underline{a}.\underline{b}$

$= 1 + 1 - 1$

$= 1$ 5

$\overrightarrow{OB} . \overrightarrow{OC} = |\overrightarrow{OB}|\,|\overrightarrow{OC}| \cos\theta$

$\underline{b}.\left(\underline{b} - \frac{1}{2}\underline{a}\right) = 1 \times 1 \times \cos\theta$

$|\underline{b}|^2 - \frac{1}{2}\underline{a}.\underline{b} = \cos\theta$

$\cos\theta = 1 - \frac{1}{2}$

$= \frac{1}{2}$

$\therefore \theta = \pi/3$ 5

$\therefore A\hat{O}B = E\hat{O}D = \pi/3$ 5

OE මගින් $A\hat{O}D$ සමච්ඡේදය කරයි.

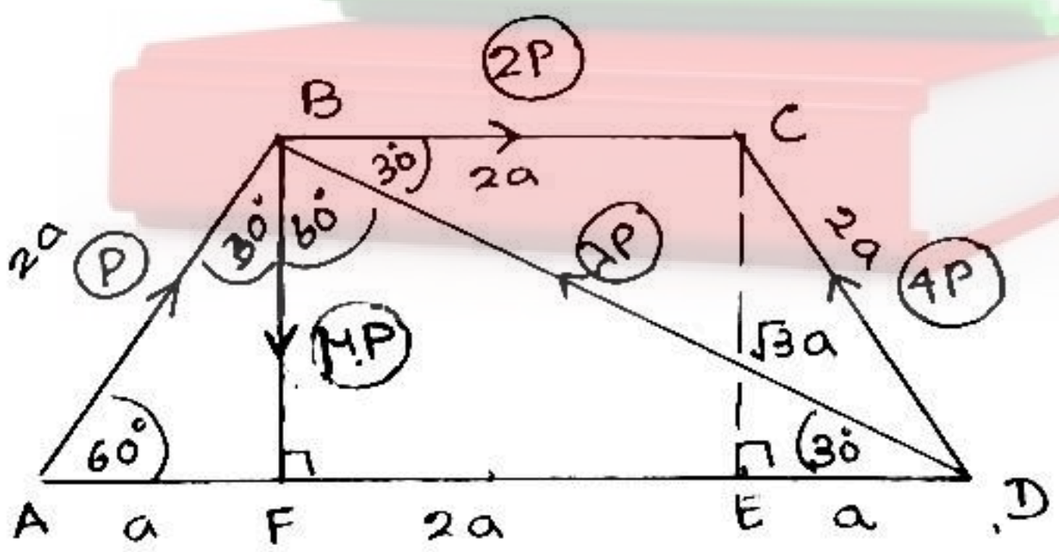
$\overrightarrow{OA}.\overrightarrow{OB} = |\overrightarrow{OA}|\,|\overrightarrow{OB}| \cos\pi/3$

$\underline{a}.\underline{b} = |\underline{a}||\underline{b}| = \frac{1}{2}$

$= 2 \cdot 1 \times \frac{1}{2} = 1$ 5

25

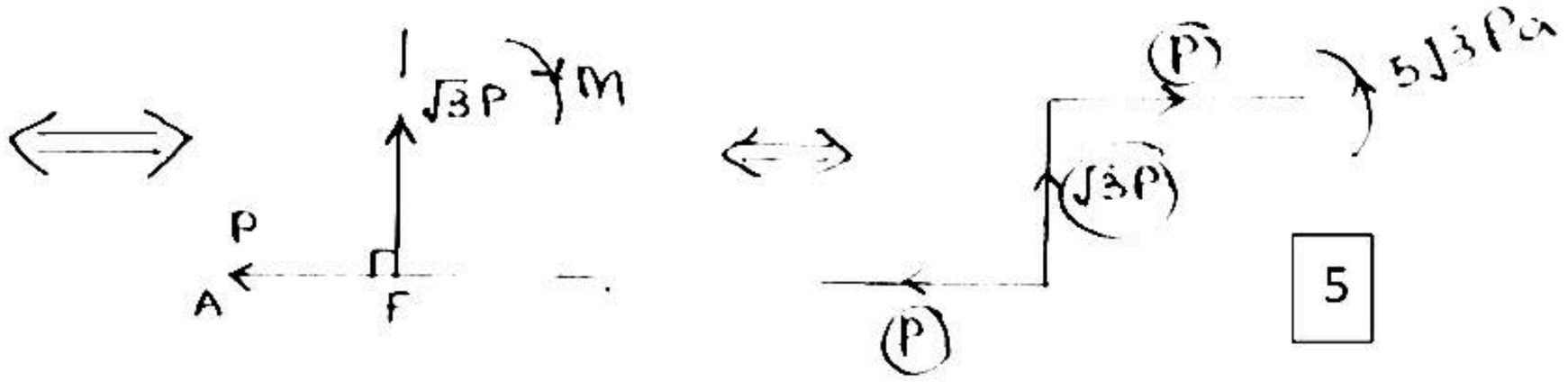
b)



$AD = 4a$

$AB = BC = 2a$

$B\hat{A}D = 60°$



5

$\leftarrow$; $P = -P\cos 60^\circ - 2P + \lambda P\cos 30^\circ + 4P\cos 60^\circ$ 10

$1 = -\frac{1}{2} - 2 + \frac{\sqrt{3}\lambda}{2} + 2$

$\frac{\sqrt{3}\lambda}{2} = \frac{3}{2} \Rightarrow \lambda = \sqrt{3}$ 5

$\uparrow$; $\sqrt{3}P = P\cos 30^\circ - \mu P + \lambda P\cos 60^\circ + 4P\cos 30^\circ$ 10

$\sqrt{3} = \frac{\sqrt{3}}{2} - \mu + \frac{\sqrt{3}}{2} + 2\sqrt{3}$

$\mu = 2\sqrt{3}$ 5

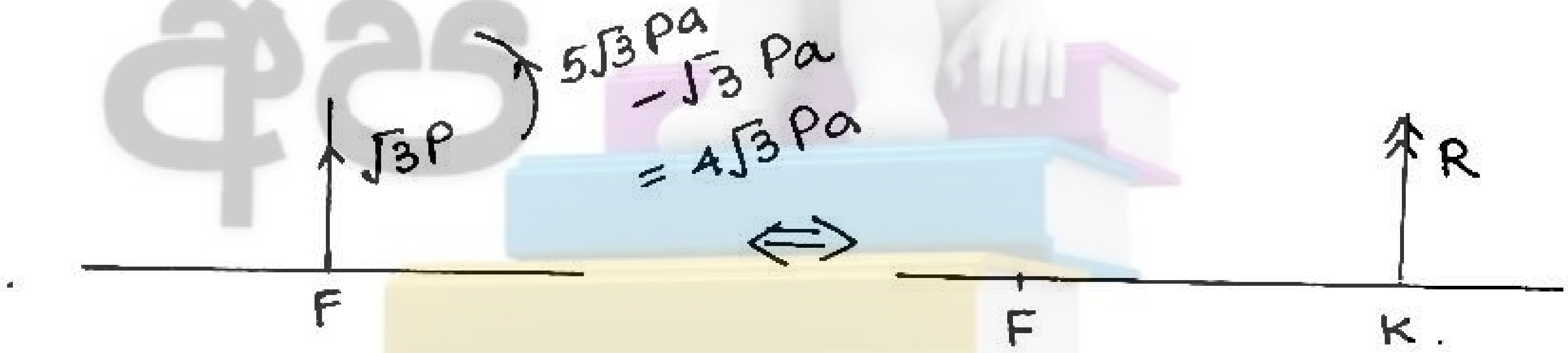
B $\circlearrowleft$ $4P\cos 30^\circ \times 2a = -M - P.\sqrt{3}a$ 10

$M = -4\sqrt{3}Pa - \sqrt{3}Pa$

$= -5\sqrt{3}Pa$ 5

$\therefore$ යුග්මය $5\sqrt{3}Pa \circlearrowleft$ වේ. 5 50

5

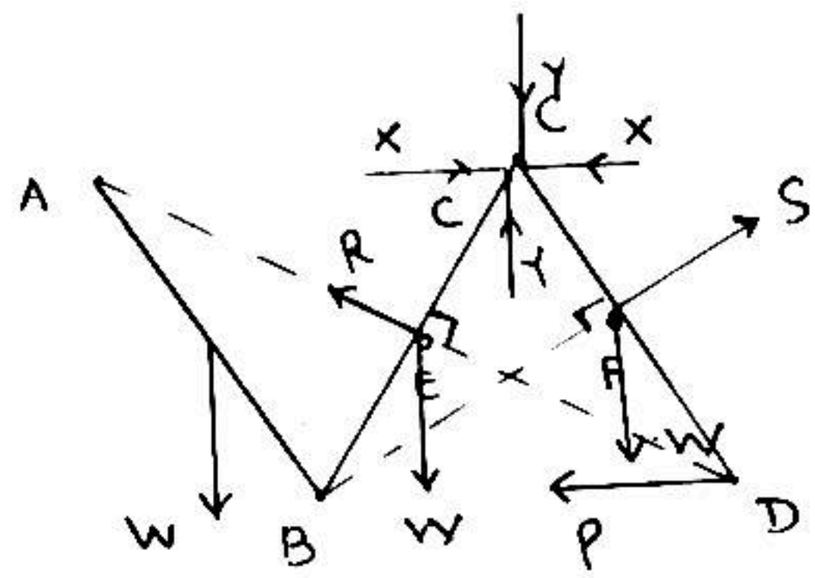


$R = \sqrt{3}P$ 5

F $\circlearrowleft$; $4\sqrt{3}Pa = R.FK$.

$FK = \frac{4\sqrt{3}Pa}{\sqrt{3}P} = \underline{\underline{4a}}$ 5 10

(15) a)



පද්ධතිය A)

$S.2a = P.2a.\cos 30^\circ + W(a+3a+5a)\cos 60^\circ$ 5

$2Sa = P.2a.\frac{\sqrt{3}}{2} + 9Wa.\frac{1}{2}.$

$S = \frac{9W}{4} + \frac{\sqrt{3}P}{2}$ 5

CD සලකා C)

$S.a = P.2a\cos 30^\circ + W.a\cos 60^\circ$ 5

$S = \sqrt{3}P + \frac{W}{2}$ 5

$\therefore \frac{9W}{4} + \frac{\sqrt{3}P}{2} = \sqrt{3}P + \frac{W}{2}$

$\frac{\sqrt{3}P}{2} = \frac{7W}{4}$ 5

$P = \frac{7W}{2\sqrt{3}}$

$S = \frac{7W}{2} + \frac{W}{2} = 4W$ 5

ABCD සලකා B)

$R.a = W(a+3a)\cos 60^\circ$

$R = 2W$ 5

CD සලකා

$\uparrow Y + W - S\cos 60^\circ = 0$

$Y = \frac{4W}{2} - W$ 5

$= W$ 5

$\rightarrow -X - P + S\cos 30^\circ = 0$

$X = 4W\frac{\sqrt{3}}{2} - \frac{7W}{2\sqrt{3}}$ 5

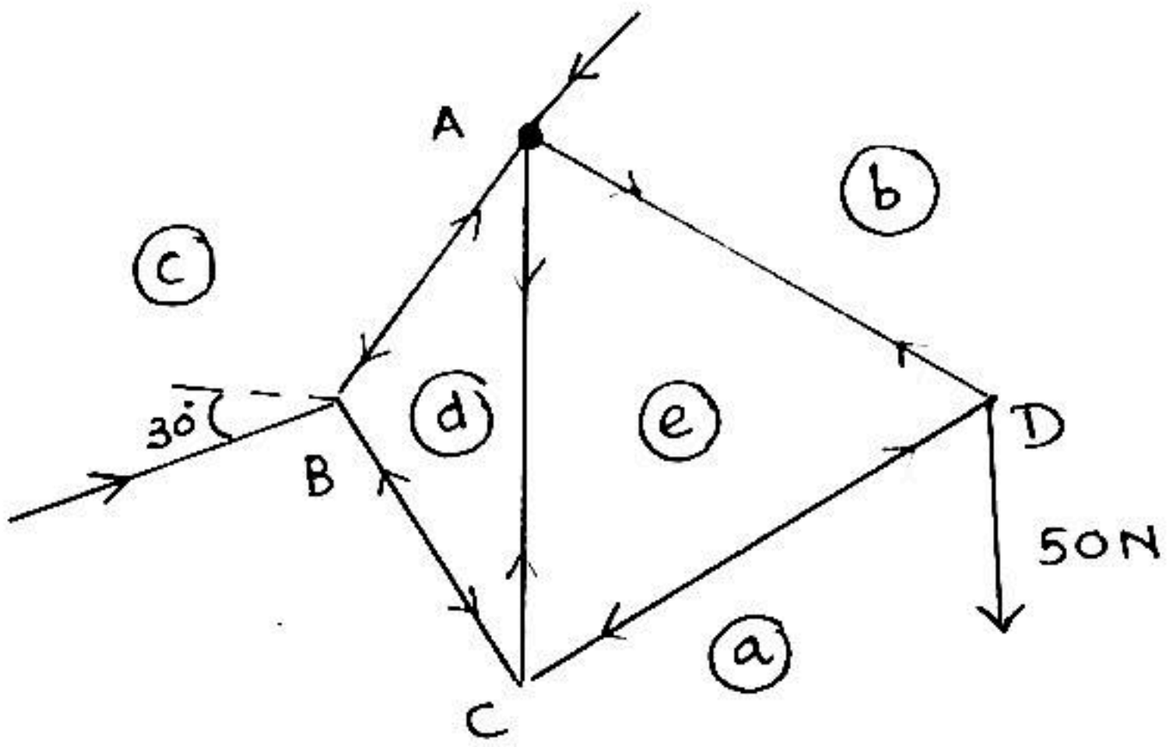
$= \frac{(12-7)W}{2\sqrt{3}}$

$= \frac{5W}{2\sqrt{3}}$

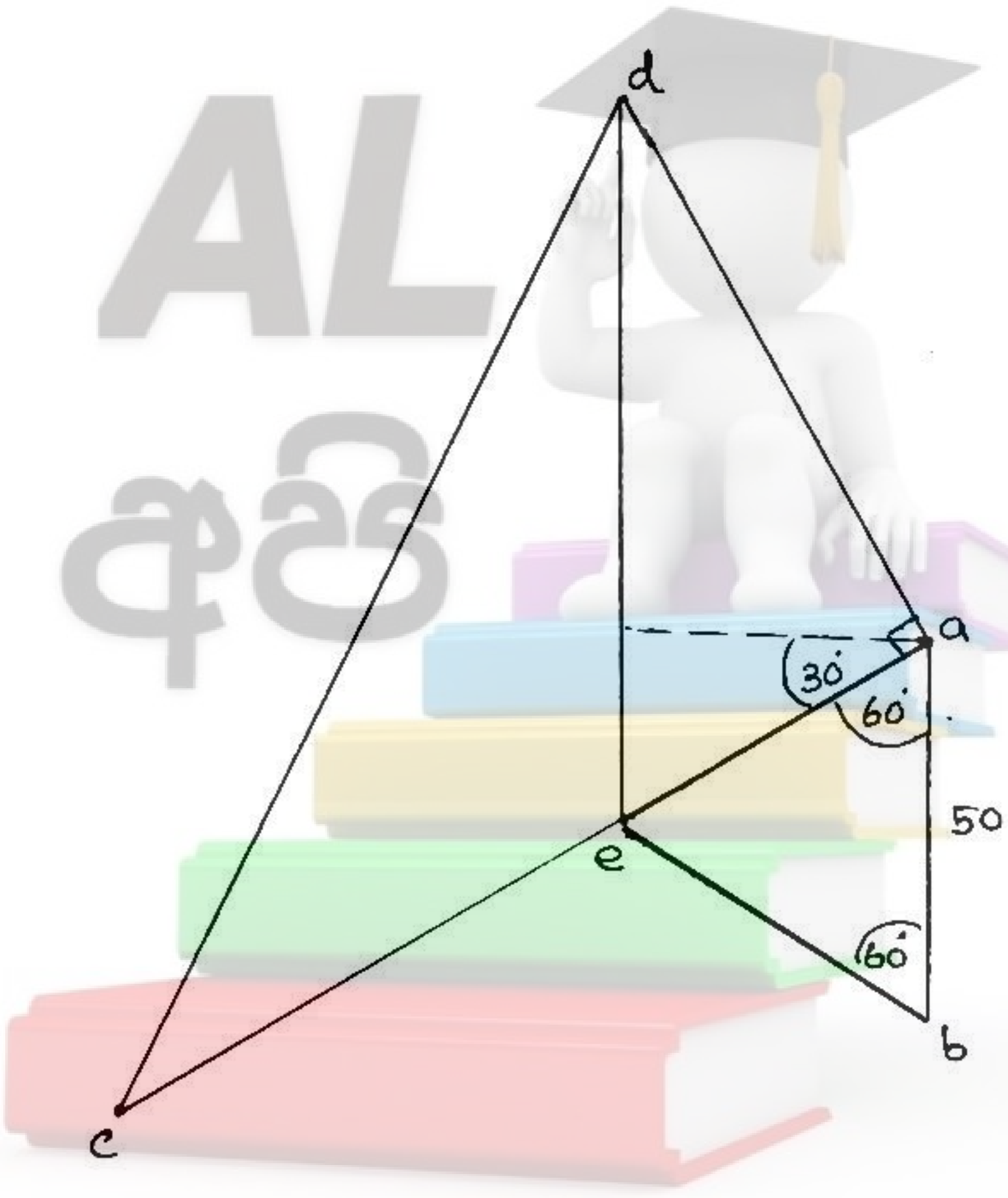
$= \frac{5\sqrt{3}W}{6}.$ 5

60

(15) b)



5



30

දණ්ඩ	තුනසා බලය	ආතති/තෙරපුම
AB (cd)	$100\sqrt{3}$ N	තෙරපුම
BC (ad)	$50\sqrt{3}$ N	තෙරපුම
CA (de)	100N	ආතති
CD (ae)	50N	තෙරපුම
DA (be)	50N	ආතති.

50

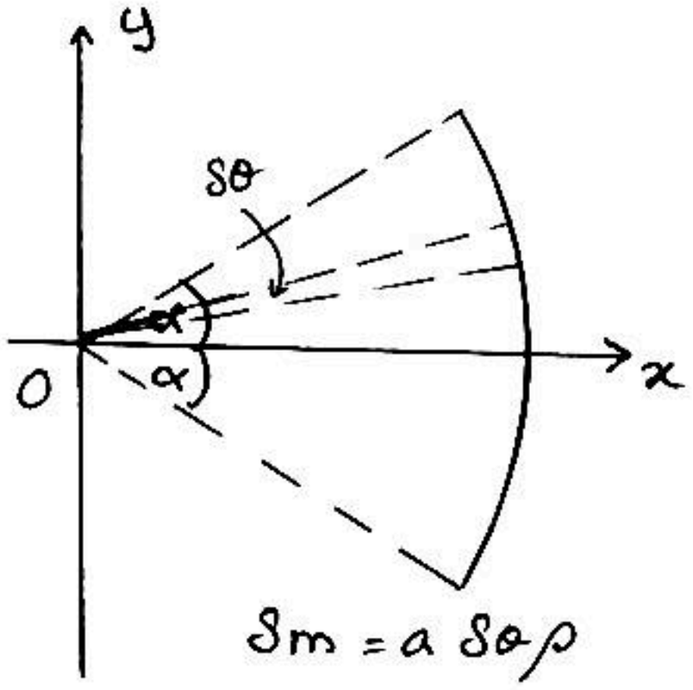
$P = Ca$

$= 100\sqrt{3} \times \frac{\sqrt{3}}{2}$

$= 150N$

10

(16)



සමමිතියෙන් ඝනාර්ධ කේතුය ox මත වේ.

O සිට $\bar{x}$ නම්,

5

$$\bar{x} = \frac{\int x\,dm}{\int dm}$$

$$= \frac{\int_{-\alpha}^{\alpha} a\,d\theta\,\rho\,a\cos\theta}{\int_{-\alpha}^{\alpha} a\,d\theta\,\rho}$$

5

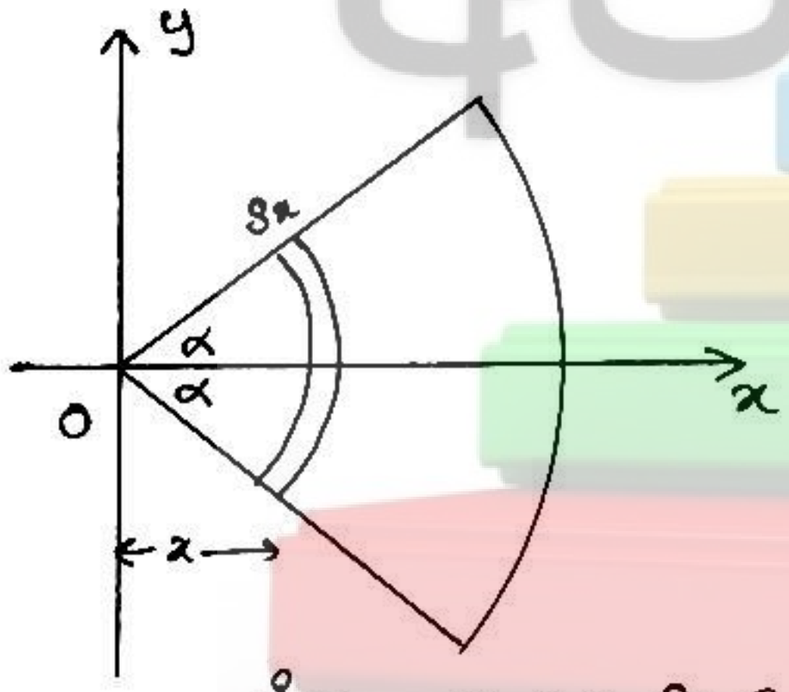
$$= \frac{a\int_{-\alpha}^{\alpha} \cos\theta\,d\theta}{\int_{-\alpha}^{\alpha} d\theta}$$

$$= \frac{a[\sin\theta]_{-\alpha}^{\alpha}}{[\theta]_{-\alpha}^{\alpha}}$$ 5

$$= \frac{a[\sin\alpha - \sin(-\alpha)]}{\alpha-(-\alpha)}$$ 5

$$= \frac{a\sin\alpha}{\alpha}$$ 5

25



සමමිතියෙන් ඝනාර්ධ කේතුය ox මත පිහිටයි. O සිට $\bar{x}$ නම්

$$\bar{x} = \frac{\int x\,dm}{\int dm}$$

$$= \frac{\int_0^a x \cdot 2\alpha\,dx\,\rho\left(\frac{x\sin\alpha}{\alpha}\right)}{\int_0^a x \cdot 2\alpha\,dx\,\rho}$$ 5

$$= \frac{\sin\alpha}{\alpha}\cdot\frac{\int_0^a x^2\,dx}{\int_0^a x\,dx}$$ 5

$$= \frac{\sin\alpha}{\alpha}\left[\frac{x^3}{3}\right]_0^a \times \left[\frac{2}{x^2}\right]_0^a$$ 5

$$= \frac{\sin\alpha}{\alpha}\cdot\frac{a^3}{3}\times\frac{2}{a^2}$$

$$= \frac{2a\sin\alpha}{3\alpha}$$ 5

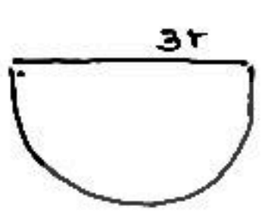
20

(6) Cont...

අර්ධ වෘත්තාකාර කාතයෙහි ස්. මේ. $\frac{a \sin \pi/2}{\pi/2} = \frac{2a}{\pi}$ 5

අර්ධ වෘත්තාකාර ආස්තරයක ස්. කේ. $\frac{2a \sin \pi/2}{3\pi/2} = \frac{4a}{3\pi}$ 5

10

වස්තුව	ස්කන්ධය ($M = \pi r^2 \rho$ · ගන්න)	ස්. මේ. 0 0B සිට සෘ →	ස්. කේ. 0 ↓ 0C සිට සෘ
	$m_1 = \frac{\pi (3r)^2 \rho}{2} = \frac{9\pi r^2 \rho}{2}$ $= \frac{9}{2} M$ 5	0 5	$\frac{4(3r)}{3\pi}$ $= \frac{4r}{\pi}$ 5
	$m_2 = \pi r \cdot (r\rho) = \pi r^2 \rho$ $= M$ 5	$3r + \frac{2r}{\pi}$ 5 $= \frac{r}{\pi}(3\pi+2)$	r 5
	$m_3 = \frac{1}{2}(2r)^2 \frac{\pi}{3} \rho$ $= \frac{2}{3} m$ 5	0 5	$\frac{2r \sin \pi/6}{3\pi/6}$ $= \frac{2r}{\pi}$ 5
	$m_4 = \frac{1}{2}(2r)^2 \frac{\pi}{3}(2\rho)$ $= \frac{4}{3} m$ 5	0 5	$= \frac{2r}{\pi}$ 5
සංයුක්ත වස්තුව	$m_1 + m_2 - m_3 + m_4$ $= \frac{m}{6}(27+6-4+8)$ $= 37m/6$ 5	$\bar{x}$	$\bar{y}$

65

ස්. කේ. අර්ථ දැක්වීමෙන්

$$\bar{x} = \frac{\sum x\, dm}{\sum dm}$$

$$= \frac{\left(\frac{9m}{2} \cdot 0\right) + m \cdot \frac{r}{\pi}(3\pi+2) - \frac{2m}{3} \cdot 0 + \frac{4m}{3} \cdot 0}{37m/6}$$ 5

$$= \frac{6r(3\pi+2)}{37\pi}$$ 5

10

(16) Cont...

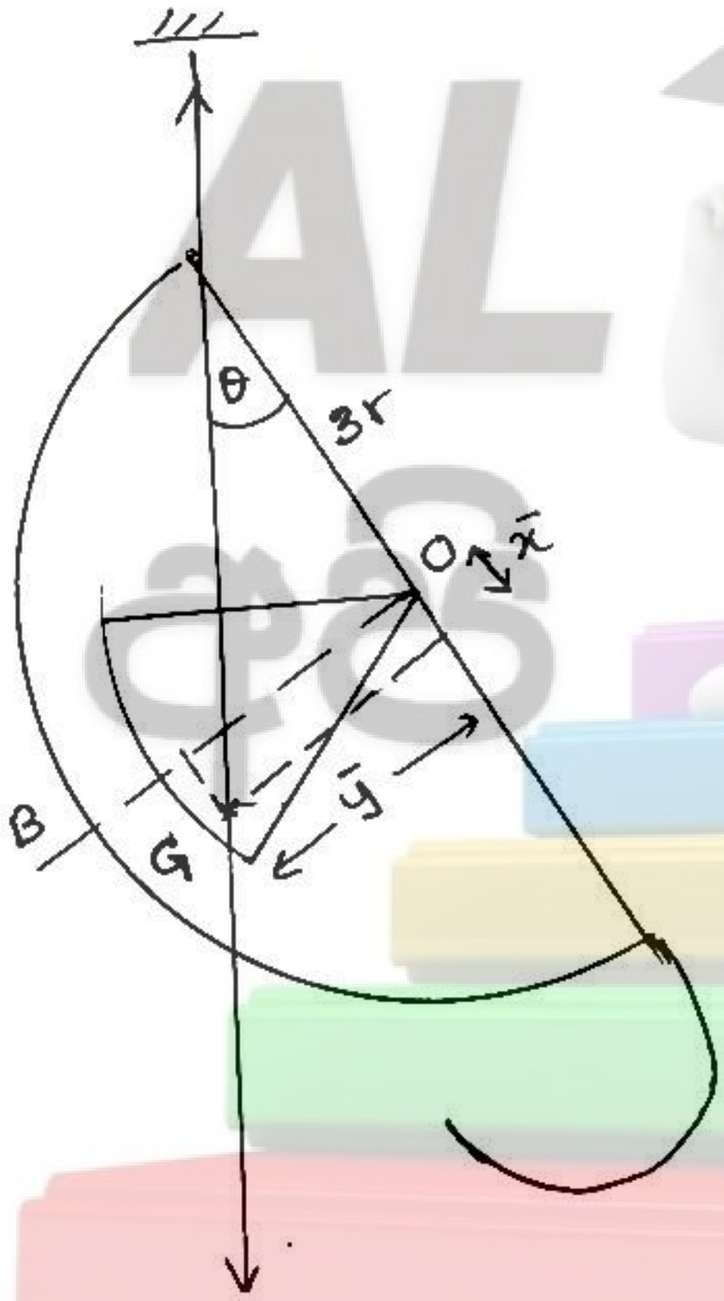
$$\bar{y} = \frac{\frac{9m}{2} \cdot \frac{4r}{\pi} + m.r - \frac{2m}{3} \cdot \frac{2r}{\pi} + \frac{4m}{3} \cdot \frac{2r}{\pi}}{37m/3}$$ 5

$$= \frac{r}{3\pi}\left[54 + 3\pi - 4 + 8\right] \times \frac{6}{37}$$

$$= \frac{2r}{37\pi}\left[58 + 3\pi\right]$$

$$= \left(\frac{58+3\pi}{37\pi}\right)r$$ 5

10



$$\tan\theta = \frac{\bar{y}}{3r + \bar{x}}$$

$$= \frac{(58+3\pi)r}{37\pi} \times \frac{37\pi}{3r + 6r(3\pi+2)}$$ 5

$$= \frac{(58+3\pi)}{(15+18\pi)}$$

$$= \frac{58+3\pi}{3(5+6\pi)}$$ 5

$$\theta = \tan^{-1}\left[\frac{58+3\pi}{3(5+6\pi)}\right]$$

10

(17) a) $P(B_1B_2) = \frac{p}{2p-1} \times \frac{p-1}{2p-2} = \frac{3}{10}$ 5

$$\frac{p}{2p-1} \times \frac{1}{2} = \frac{3}{10}$$

$$5p = 6p - 3$$

$$p = 3$$ 5

10

i) $P(B_1B_1) \times P(W_BW_B) \times [P(B_cW_c) + P(W_cB_c)]$

$= \frac{3}{5} \times \frac{2}{4} \times \frac{3}{5} \times \frac{2}{4} \times \left[\frac{2}{4} \times \frac{2}{3} + \frac{2}{4} \times \frac{2}{3}\right]$ 10

$= \frac{6}{20} \times \frac{6}{20} \times \frac{8}{12}$

$= \frac{3}{50}$ 5

15

ii) $P(B_AB_A) \times P(B_BB_B) \times [P(B_cB_c)]$

$= \frac{3}{5} \times \frac{2}{4} \times \frac{2}{5} \times \frac{1}{4} \times 1$ 10

$= \frac{3}{100}$ 5

15

iii) $P(B_AW_A) \times P(B_BW_B) \times [P(B_cW_c) + P(W_cB_c)]$

$= \frac{3}{5} \times \frac{2}{4} \times \frac{2}{5} \times \frac{3}{4} \times \left[\frac{2}{4} \times \frac{2}{3} + \frac{2}{4} \times \frac{2}{3}\right]$ 10

$= \frac{6}{20} \times \frac{6}{20} \times \frac{8}{12}$

$= \frac{3}{50}$ 5

15

b) $\bar{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$ 5 $\qquad \sigma_x^2 = \dfrac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{\sum_{i=1}^{n} f_i}$

$$= \frac{\sum f_i x_i^2}{\sum f_i} - \bar{x}^2 \quad \boxed{5}$$

$$d_i = x_i - A \implies x_i = d_i + A$$

$$\bar{x} = \frac{\sum f_i (d_i + A)}{\sum f_i} = \frac{\sum f_i d_i}{\sum f_i} + A \frac{\sum f_i}{\sum f_i}$$

$$= A + \frac{\sum f_i d_i}{\sum f_i} \quad \boxed{5}$$

$$\sum f_i x_i^2 = \sum f_i (A + d_i)^2 = \sum f_i (A^2 + 2Ad_i + d_i^2)$$

$$= A^2 \sum f_i + 2A \sum f_i d_i + \sum f_i d_i^2 \ .$$

$$\bar{x}^2 = \left(A + \frac{\sum f_i d_i}{\sum f_i} \right)^2$$

$$= A^2 + 2A \frac{\sum f_i d_i}{\sum f_i} + \left(\frac{\sum f_i d_i}{\sum f_i} \right)^2 \quad \boxed{5}$$

$$\sigma_x^2 = A^2 + 2A \frac{\sum f_i d_i}{\sum f_i} + \frac{\sum f_i d_i^2}{\sum f_i} - A^2 - 2A \frac{\sum f_i d_i}{\sum f_i}$$

$$- \left(\frac{\sum f_i d_i}{\sum f_i} \right)^2$$

$$= \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i} \right)^2 \quad \boxed{5}$$

25

බර	සංඛ්‍යාතය f_i	මධ්‍ය අගය x_i	$x_i - 80$ d_i	$f_i d_i$	$f_i d_i^2$
45-55	x	50	-30	$-30x$	9000
55-65	x	60	-20	$-20x$	4000
65-75	$2x$	70	-10	$-20x$	2000
75-85	$5y$	80	0	0	0
85-95	$3x$	90	10	$30x$	3000
95-105	y	100	20	$20y$	2000
	100			-300	20000

5 5

10

$\sum f_i = 100$

$7x + 6y = 100$ — (1)

5

$\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i}$

$77 = 80 + \frac{-40x + 20y}{100}$

$40x - 20y = 300$

$2x - y = 15$ — (2)

5

(1) + 6 × (2)

$19x = 190$

$x = 10$

$y = 5$

5

15

$\sigma^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2$

$= \frac{20000}{100} - \left(\frac{300}{100}\right)^2$ 5

$= \frac{2000000 - 90000}{100^2}$

$= \frac{1910000}{10000}$

$= 191$ 5

10

මාතය $= L + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) C$

$= 85 + \left(\frac{5}{25+5}\right) \times 10$ 5

$= 85 + \frac{5}{3}$

$= 86.6$ 5

10

මධ්‍යස්ථය $= L + \left(\frac{n/2 + f_c}{f}\right) \times C$

$= 75 + \left(\frac{50 - 40}{25}\right) \times 10$ 5

$= 75 + 4$

$= 79$ 5

10

$\sigma = \sqrt{191} = 13.82$ 5

05

	f_i	d_i	$f_i d_i$
35 - 45	10	-30	-300
45 - 55	10	-20	-200
55 - 65	20	-10	-200
65 - 75	25	0	0
75 - 85	30	10	300
85 - 95	05	20	100

5

$\bar{x} = A' + \frac{\Sigma f_i d_i}{\Sigma f_i}$

$= 70 + \frac{(-300)}{100}$

$= 70 - 3$

$= 67$

5

10